University of Saskatchewan
Department of Mathematics & Statistics

Time: 3 hours — Mathematics 124.3 (2,4,6,8) Final Examination — April 17, 2002
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No books, notes or calculators are allowed.

- This examination is in two parts.
- Solutions to Part A are to be written out in the booklet provided.
- Solutions to Part B are to be entered on the opscan sheet. There is no penalty for an incorrect response to a question in Part B.

Each question in Part A is worth five marks.
Question B1 is worth five marks.
Question B2 is worth four marks.
Questions B3–B9 are worth three marks each.
Questions B10–B14 are worth two marks each.

Part A

A1. Resolve $\dfrac{2}{x^3 - x^2}$ into partial fractions.

A2. Evaluate $\displaystyle\int \sec 2x \tan^3 2x \, dx.$

A3. Evaluate $\displaystyle\int \frac{2 + 3\cos x}{\sin x} \, dx.$

A4. Evaluate $\displaystyle\int e^{\sqrt{x}} dx.$

A5. Evaluate $\displaystyle\int \frac{e^x + 1}{e^x - 1} \, dx.$

A6. Evaluate $\displaystyle\int \frac{dx}{\sqrt{2x - x^2}}.$

A7. Evaluate $\displaystyle\int_{-\infty}^{-2} \frac{2dx}{x^2 - 1}.$

A8. A cylindrical drum of height 1 m and diameter 60cm contains water to a depth of 80 cm. Calculate the work done in pumping one half of the water to a height of 10 cm above the top of the drum. (The density of water is 1000 kg/m^3.)

A9. Determine the coordinates of the centroid of the region bounded by $y = e^x$, the x axis and the ordinates $x = 0$, $x = 1$.

A10. Use the method of cylindrical shells (NO OTHER METHOD IS ACCEPTABLE) to determine the volume of a right circular cone whose vertical height is 6 and base radius is 2.

A11. The region bounded by the graph of $y = e^x \sin x$, the x axis and the ordinates $x = 1$, $x = \pi/2$ is rotated about the x axis to form a solid, S. Set up the integral which represents the curved surface area of S. Do not attempt to solve the integral.

A12. A radioactive substance has a half-life of 100 days. If a sample has an initial mass of 200g, determine

(i) the mass after 60 days.

(ii) the time for the mass to be reduced to 70 g.

Part B

The choices for question B1 are

(a) $\pi\left(\frac{4-\pi}{4}\right)$ (b) $\pi\left(\frac{8-\pi}{4}\right)$ (c) $\pi\left(\frac{16-\pi}{4}\right)$ (d) $\pi\left(\frac{32-\pi}{4}\right)$

(e) $\pi\left(\frac{4-\pi}{2}\right)$ (f) $\pi\left(\frac{8-\pi}{2}\right)$ (g) $\pi\left(\frac{16-\pi}{2}\right)$ (h) $\pi\left(\frac{32-\pi}{2}\right)$

(i) $\pi\left(\frac{8+\pi}{4}\right)$ (j) $\pi\left(\frac{32+\pi}{4}\right)$

B1. The region bounded by the y axis, the curve $y = \cos x$ and the x axis between $x = 0$ and $x = \frac{\pi}{2}$ is rotated about the line $y = 2$ to form a solid. Determine the volume of the solid.

The choices for question B2 are

(a) 5 (b) 5.5 (c) 6 (d) 6.5 (e) 7

(f) 7.5 (g) 8 (h) 8.5 (i) 9 (j) 9.5

B2. Compute the area bounded by the line $y = 3x$ and the curve $y = x(x^2 - 1)$.

The choices for questions B3–B7 are

(a) 0	(b) 1	(c) 2	(d) 4	(e) 5
(f) 6	(g) 7	(h) 8	(i) e	(j) ∞

B3. Evaluate $\lim_{x\to 0} \dfrac{e^x - 1 + 4x}{x^2 + x}$.

B4. Evaluate $\lim_{x\to 0+} \left(\dfrac{1}{x^3} - \dfrac{1}{x^2}\right)$.

B5. Evaluate $\lim_{x\to\infty} (e^x + 2x)^{1/x}$.

B6. Use the trapezoidal rule with $n = 3$ to determine an approximate value for $\displaystyle\int_0^9 \frac{dx}{1+x}$.

Write your answer in decimal form.
What is the digit immediately to the right of the decimal point?

B7. Use Simpson's rule with $n = 4$ to determine an approximate value for $\displaystyle\int_0^8 \frac{dx}{1+x}$.

Write your answer in decimal form.
What is the digit immediately to the right of the decimal point?

The choices for question B8 are

(a) $\ln 2$	(b) $\ln 3$	(c) $\dfrac{1}{\ln 2}$	(d) $\dfrac{2}{\ln 2}$	(e) $\dfrac{3}{\ln 2}$
(f) $\dfrac{4}{\ln 2}$	(g) $\dfrac{1}{\ln 3}$	(h) $\dfrac{2}{\ln 3}$	(i) $\dfrac{3}{\ln 3}$	(j) $\dfrac{4}{\ln 3}$

B8. Calculate the average value of the function e^x on the interval $[\ln 3, \ln 6]$.

The choices for question B9 are

(a) .703	(b) .708	(c) .711	(d) .713	(e) .718
(f) .721	(g) .723	(h) .728	(i) .733	(j) .738

B9. Use differentials to estimate $\ln(2.06)$ given that $\ln 2 \approx .693$.

The choices for question B10 are

(a) 0	(b) $\frac{1}{2}$	(c) $\frac{1}{5}$	(d) $\frac{1}{15}$	(e) 2
(f) 5	(g) 15	(h) 12	(i) 30	(j) 36

B10. If $\tanh x = \frac{4}{5}$ determine the value of $3\cosh x$.

The choices for questions B11 and B12 are

(a) 1	(b) -1	(c) $\frac{\pi}{2}$	(d) $-\frac{\pi}{2}$	(
(f) $-\pi$	(g) $\frac{1}{\sqrt{2}}$	(h) $-\frac{1}{\sqrt{2}}$	(i) $\sqrt{2}$	(

B11. Evaluate the derivative of $\sinh^{-1} x$ at $x = 1$.

B12. Evaluate $2 \lim_{x\to\infty} \tan^{-1} x$.

The choices for questions B13 and B14 are

(a) 0	(b) $\frac{1}{2}$	(c) 1	(d) 2
(f) e	(g) $\frac{e^2+1}{e}$	(h) $\frac{e}{e^2+1}$	(i) $e(e^2+1)$

B13. Evaluate $\frac{1+\tanh x}{1-\tanh x}$ at $x = \frac{1}{2}$.

B14. Evaluate the derivative of $\tan^{-1}(e^x)$ at $x = 1$.

The End